Heterotic Line Bundle Models

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Success consists of going from failure to failure without loss of enthusiasm. – Winston Churchill

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- There are great advantages of the line bundle construction, leading to constraints on operators.

I shall discuss one example:

the tetraquadric hypersurface

Line Bundles on Smooth Calabi-Yau 3-folds

The line bundle construction of $E_8 \times E_8$ heterotic string models follows three stages:

1. $\mathcal{N} = 1$, 4d GUT models with gauge group G, from $E_8 \to G \times H$;
2. break G to G_{SM} and check for the right spectrum
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N.B. All this is possible in a systematic way by scanning over a huge set of models and filtering out the unsuitable ones. No geometric engineering.

Line Bundle Models

The line bundle programme has (so far) led to:

- about 44,000 $\mathcal{N} = 1$, 4d GUT models with:
  - gauge group $SU(5) \times U(1)^4$ (the extra $U(1)$s are generically anomalous and have massive gauge bosons);
  - matter spectrum in $\mathbf{10}$ and $\bar{\mathbf{5}}$; correct number of families;
  - one or several $\mathbf{5} - \bar{\mathbf{5}}$ pairs;
  - no exotic fields
  - other features which make the doublet-triplet splitting problem easy to overcome
- a lot more models after breaking the GUT group to $G_{SM} \times U(1)^4$

GUT Models from Line Bundles

The line bundle construction of heterotic string models follows three stages:

1. $\mathcal{N} = 1$, 4d GUT models with gauge group H, from $E_8 \to G \times H$.
   Ingredients:

- a smooth Calabi-Yau three-fold X;
- a holomorphic vector bundle $V \to X$ with structure group $G \subset E_8$.
  Traditionally: $G = SU(n)$ for $n = 3, 4, 5$.

  Here V is a sum of 5 line bundles $V = \bigoplus_{a=1}^{5} L_a$. $G = U(1)^5$.
- certain requirements on X and V.

GUT Models from Line Bundles

Requirements on X and V:

- $c_1(V) = \bigoplus_{a=1}^{5} c_1(L_a) = 0$. Hence $G = S(U(1)^5) \cong U(1)^4$ and then the GUT group is $H = SU(5) \times S(U(1)^5) \cong SU(5) \times U(1)^4$.
- anomaly cancellation: $c_2(TX) - c_2(V) = [\text{Eff.Curve}]$. Hence $c_2(TX) \geq c_2(V)$
- $\mathcal{N} = 1$ supersymmetry implies that the gauge connection on V satisfies the hermitian YM equations.

  By the Donaldon-Uhlenbeck-Yau theorem this is possible if and only if V has vanishing slope and is polystable.

Slope-stability of vector bundles:

- slope of a vector bundle V defined as:

$$\mu(V) = \frac{1}{\mathrm{rk}\, V}\int_X c_1(V)\wedge J\wedge J = \frac{1}{\mathrm{rk}\, V}\sum_{r,s,t=1}^{h^{1,1}(X)} d_{rst}\, c_1^r(V)t^s t^t$$

  where $J = t^r J_r$ is the Kähler form on X; t^r are Kähler moduli

- a bundle is stable if $\mu(\mathcal{F}) < \mu(V)$ for any coherent sub-sheaf $\mathcal{F}\subset V$ with $0 < \mathrm{rk}(\mathcal{F}) < \mathrm{rk}(V)$; a bundle is poly-stable if it can be written as a direct sum of stable bundles $V = \bigoplus_a V_a$ with $\mu(V) = \mu(V_a)$, for all a

- slope-stability is a moduli-dependent question

- for a line bundle $\mathrm{rk}(L) = 1$, stability criterion is trivially true

- for a sum of line bundles $V = \bigoplus_a L_a$: $\mu(L_a) = 0$ simultaneously for all a somewhere in the interior of the Kähler cone.

SM Gauge Group and Spectrum

The line bundle construction of heterotic string models follows three stages:

1. $\mathcal{N} = 1$, 4d GUT models with gauge group G, from $E_8 \rightarrow G \times H$;
2. break G to G_{SM} and check for the right spectrum
   Ingredients:

   – need non-trivial $\pi_1(X)$; solution: quotient X by the free action of a discrete group $\Gamma \rightarrow X$;

   – ensure that there exists an action of Γ on V so that V induces a bundle $\tilde{V} \rightarrow X/\Gamma$ (equivariant structure on V);

   – complete the bundle $\tilde{V} \rightarrow X/\Gamma$ with a Wilson line to break the GUT group to G_{SM}.

Operators

The line bundle construction of heterotic string models follows three stages:

1. $\mathcal{N} = 1$, 4d GUT models with gauge group G, from $E_8 \to G \times H$;
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3. constrain the 4d supergravity operators

The $SU(5)$ multiplets (and the G_{SM} multiplets) come with certain patterns of charges under the extra $U(1)$s. Using these charges, one can ensure things like proton stability or R-parity conservation.

Choosing the Manifold

The class of Calabi-Yau 3-folds realised as complete intersections in products of projective spaces (CICYs) form a particularly suitable set for supporting the line bundle construction:

- the class is relatively small (7890 configuration matrices);
- there is a classification of linearly realised freely acting discrete symmetries [Candelas, Davies 2008; Braun, 2010];
- cohomology computations of line bundles on CICYs are largely possible [Anderson, He, Lukas, 2008];

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We selected from the list of CICY (as constructed by Candelas, Lütken and Shimmrick) those which:

- figure in Braun's list of discrete symmetries
- are favourable (i.e. their second cohomology descends from that of the embedding product of projective spaces)

Choosing the Manifold

In this way we end up with 71 manifolds:

- $h^{1,1}(X) = 2$: 6 manifolds
- $h^{1,1}(X) = 3$: 12 manifolds
- $h^{1,1}(X) = 4$: 19 manifolds
- $h^{1,1}(X) = 5$: 23 manifolds
- $h^{1,1}(X) = 6$: 8 manifolds

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Each manifold has smooth quotients by one or more discrete groups, sometimes with different orders.

Spectrum and Index Requirements

The matter spectrum is given by the following cohomologies:

- **10** multiplets: $H^1(X, V) = \bigoplus_a H^1(X, L_a)$
- $\bar{\mathbf{5}}$ multiplets: $H^1(X, \wedge^2 V) = \bigoplus_{a<b} H^1(X, L_a \otimes L_b)$
- **5** multiplets: $H^2(X, \wedge^2 V) \cong H^1(X, \wedge^2 V^*) = \bigoplus_{a<b} H^1(X, L_a^* \otimes L_b^*)$
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Require:

- $h^1(X,V) = 3|\Gamma|$ and $h^1(X,V^*) = 0$:
  3 $SU(5)$ **10** families and no $\overline{\mathbf{10}}$s after quotienting by Γ
- $h^1(X,\wedge^2 V) - h^1(X,\wedge^2 V^*) = 3|\Gamma|$:
  chiral asymmetry of 3 $\bar{\mathbf{5}}$s after quotienting

The Scan

A line bundle is given by a set of integers

$$L = \mathcal{O}_X(\vec{k}) = \bigotimes_\alpha \mathcal{O}_{\mathbb{P}^{n_\alpha}}(k_\alpha)|_X$$

for $X \subset \prod_\alpha \mathbb{P}^{n_\alpha}$. A sum of 5 line bundles is then given by a matrix of integers with $h^{1,1}(X)$ rows and 5 columns.

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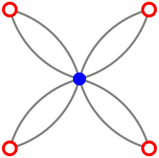
$\sim 10^{40}$ such matrices and selected $\sim 44,000$ models which lead to consistent $SU(5)$ GUTs.

The Tetraquadric Hypersurface

$$\mathcal{Q}^{4,68} = \begin{matrix} \mathbb{P}^1 \\ \mathbb{P}^1 \\ \mathbb{P}^1 \\ \mathbb{P}^1 \end{matrix} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}^{4,68}$$



- The manifold $\mathcal{Q}^{4,68}$ has smooth quotients by free (linear) actions of discrete groups of orders $2, 4, 8$ and 16 [Candelas, Davies 2008; Braun, 2010]

$$\mathbb{Z}_2;\ \mathbb{Z}_2 \times \mathbb{Z}_2,\ \mathbb{Z}_4;\ \mathbb{Z}_2 \times \mathbb{Z}_4,\ \mathbb{Z}_8,\ \mathbb{H};$$

$$\mathbb{Z}_4 \times \mathbb{Z}_4,\ \mathbb{Z}_4 \rtimes \mathbb{Z}_4,\ \mathbb{Z}_8 \times \mathbb{Z}_2,\ \mathbb{Z}_8 \rtimes \mathbb{Z}_2,\ \mathbb{H} \times \mathbb{Z}_2$$

Line Bundle Models on the Tetraquaric

The number of models on the tetraquadric threefold satisfying the above criteria:

$$\begin{aligned}
\{7862, 2\} &\longrightarrow \{19, 32, 35, 35, 35, 35, 35, 35, 35\} \\
\{7862, 4\} &\longrightarrow \{34, 100, 111, 115, 115, 115, 115, 115, 115\} \\
\{7862, 8\} &\longrightarrow \{17, 132, 183, 194, 199, 201, 201, 201, 201\} \\
\{7862, 16\} &\longrightarrow \{1, 5, 10, 16, 22, 22, 24, 24, 24\}
\end{aligned}$$

A Finiteness Result

In all the cases that we looked at, when we required that:

- the bundle $V \to X$ is poly-stable
- the index of the bundle V is fixed (3 times the order of Γ)
- there is an upper bound on $c_2(V)$, coming from the anomaly cancellation condition

we came to the conclusion that the number of such bundles is finite, i.e. increasing k_{max} does not produce any new models.

Conjecture: for a given Chern class, the set of line bundle sums that are poly-stable somewhere in the positive Kähler cone is finite.

We have a good understanding why this should be the case in the interior of the Kähler cone, but things get tricky at the boundary.

Final Remarks

Returning to wider picture, let me note a few points:

- The current scan is largely an experimental work; so far we have collected the data - a lot of work is required in order to fully analyse these models.
- There is an immediate challenge: improving the line bundle cohomology algorithm.
- Can we obtain an up Yukawa matrix of rank 1? In a previous scan, rank Y_u was $0, 2, 3$.
- How far into phenomenology can we push the line bundle models? (e.g. neutrino physics)