

Baez–Muniain Exercise 1: This is just basic vector calculus: Since \mathbf{E}, \mathbf{k} are constant, and $\mathbf{E} \cdot \mathbf{k} = 0$, we have

$$\nabla \cdot \mathbf{E}e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})} = \mathbf{E} \cdot \nabla e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})} = i(\mathbf{E} \cdot \mathbf{k})e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})} = 0$$

Further, wasting some time, then checking out the errata at <http://math.ucr.edu/home/baez/errata.html>, and using $i\mathbf{k} \times \mathbf{E} = \omega\mathbf{E}$ instead the relation stated in Exercise 1,

$$\nabla \times (e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})}\mathbf{E}) = (\nabla e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})}) \times \mathbf{E} = i\mathbf{k}e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})} \times \mathbf{E} = \omega\mathbf{E}e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})} = i \frac{\partial \mathbf{E}e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})}}{\partial t}$$

We haven't use the condition $\omega = |\mathbf{k}|$. However, it follows easily from the stated relations

$$\mathbf{k} \cdot \mathbf{E} = 0 \quad i\mathbf{k} \times \mathbf{E} = \omega\mathbf{E}$$

that $\omega^2 = |\mathbf{k}|^2$.