## Exercise 1, p9

Given $\underline{\xi}(t, \underline{x}) = \underline{E}\, e^{-i(\omega t - \underline{k}\cdot\underline{x})}$

Then:

① $\nabla \cdot \underline{\xi} = \frac{\partial \xi_1}{\partial x_1} + \frac{\partial \xi_2}{\partial x_2} + \frac{\partial \xi_3}{\partial x_3}$

where $\underline{\xi} = (\xi_1, \xi_2, \xi_3)$

$\underline{x} = (x_1, x_2, x_3)$

$\xi_1 = E_1\, e^{-i(\omega t - \underline{k}\cdot\underline{x})}$ where $\underline{E} = (E_1, E_2, E_3)$

$\frac{\partial \xi_1}{\partial x_1} = E_1\, e^{-i(\omega t - \underline{k}\cdot\underline{x})} \cdot (ik_1)$

where $\underline{k} = (k_1, k_2, k_3)$

and similarly for $\frac{\partial \xi_2}{\partial x_2}$, $\frac{\partial \xi_3}{\partial x_3}$.

$\nabla \cdot \underline{\xi} = ie^{-i(\omega t - \underline{k}\cdot\underline{x})}(k_1 E_1 + k_2 E_2 + k_3 E_3)$

$= ie^{-i(\omega t - \underline{k}\cdot\underline{x})} \cdot (\underline{k}\cdot\underline{E})$

$= 0$

② $\nabla \times \underline{\xi} = \nabla \times \underline{E}\, e^{-i(\omega t - \underline{k}\cdot\underline{x})}$

$= \nabla e^{-i(\omega t - \underline{k}\cdot\underline{x})} \times \underline{E} + e^{-i(\omega t - \underline{k}\cdot\underline{x})} \nabla \times \underline{E}$ ($\nabla \times \underline{E} = 0$)

$= \left(e^{-i(\omega t - \underline{k}\cdot\underline{x})} \cdot i\underline{k}\right) \times \underline{E}$

$= ie^{-i(\omega t - \underline{k}\cdot\underline{x})} \cdot (\underline{k} \times \underline{E})$

$= ie^{-i(\omega t - \underline{k}\cdot\underline{x})} (-i\omega \underline{E})$

$= \omega e^{-i(\omega t - \underline{k}\cdot\underline{x})}\, \underline{E}$

$$i \frac{\partial \underline{\mathcal{E}}}{\partial t} = i \underline{E} e^{-i(\omega t - \underline{k} \cdot \underline{x})} \cdot (-i\omega)$$

$$= \omega \underline{E} e^{-i(\omega t - \underline{k} \cdot \underline{x})}$$

So $$\nabla \times \underline{\mathcal{E}} = i \frac{\partial \underline{\mathcal{E}}}{\partial t}$$