Baez and Muniain

Exercises 72–79

72. Clear:
$$\star F_+ = \star \tfrac{1}{2}(F + \star F)$$
$$= \tfrac{1}{2}(\star F + F)$$
$$= F_+$$
and similarly for F_-.

73. Set
$$F_+ := \tfrac{1}{2}(iF + \star F)$$
$$F_- := \tfrac{1}{2}(F + i\star F)$$
Then
$$\star F_+ = \tfrac{1}{2}(i\star F - F)$$
$$= iF_+$$
and
$$\star F_- = \tfrac{1}{2}(\star F - iF)$$
$$= -iF_-\,.$$

74. Clearly equivalent:
$$\star(\text{1st equation}) = \text{2nd equation}$$
and vice versa. ($\star$ is an invertible map).

To obtain $\star_s E = iB$, we need

$$*(E_x\,dx + E_y\,dy + E_z\,dz) = i(B_x\,dy\wedge dz + B_y\,dz\wedge dx + B_z\,dx\wedge dy)$$

i.e. $iB_x = E_x$, etc., or in vector form, $\vec{B} = -i\vec{E}$. (2)

75. $E = \vec{E}_0 e^{ik_\mu x^\mu}$, $B = \vec{B}_0 e^{ik_\mu x^\mu}$

$$\partial_t B + d_s E = 0$$

$$\Rightarrow \left(ik_0 \vec{B}_0 e^{ik_\mu x^\mu} - \vec{E}_0 \wedge d_s e^{ik_\mu x^\mu}\right) = 0$$

$$\Rightarrow \left(ik_0 \vec{B}_0 - i\vec{E}_0 \wedge {}^3k\right) e^{ik_\mu x^\mu} = 0$$

$$\Rightarrow {}^3k \wedge \vec{E}_0 = -k_0 \vec{B}_0$$

$\boxed{\text{notice sign error in question!}}$

76. In terms of 3d vectors, we can write the equation

$${}^3k \wedge \tilde{E} = -i\omega *_s \tilde{E} \qquad (\omega = k^0)$$

$\boxed{\text{note sign}}$

as

$$\vec{k} \times \vec{E} = -i\omega \vec{E} \quad \dots \quad (*)$$

We can break $\vec{E}$ up into real and imaginary components:

$$\vec{E} = \vec{e} + i\vec{b}$$

Then (*) becomes the two equations

$$\vec{k} \times \vec{e} = +\omega \vec{b} \quad \dots (1)$$

$$\vec{k} \times \vec{b} = -\omega \vec{e} \quad \dots (2)$$

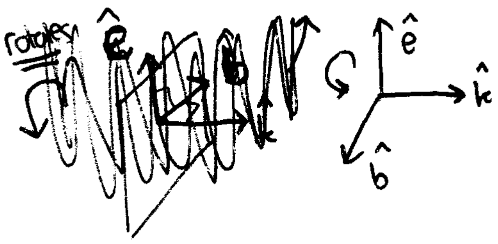
These equations have <u>directional</u> and <u>magnitude</u> content, since they can be written in terms of unit vectors as

$$\frac{|\vec{k}|\,|\vec{e}|}{\omega\,|\vec{b}|}\;\hat{k}\times\hat{e} = +\hat{b} \qquad \ldots \quad (1')$$

$$\frac{|\vec{k}|\,|\vec{b}|}{\omega\,|\vec{e}|}\;\hat{k}\times\hat{b} = -\hat{e} \qquad \ldots \quad (2')$$

which amounts to <u>three</u> pieces of information:

$$\hat{k}\times\hat{e} = +\hat{b}\,, \qquad |\vec{k}| = \omega \qquad \text{and} \qquad |\vec{e}| = |\vec{b}|.$$



this says $k_\mu k^\mu = 0$, since that is equivalent to $\omega^2 = |k|^2$.

clear. what this means

" b chases e "

77. We have $E = \tilde{E}\, e^{i k_\mu x^\mu}$ which translates into

$$E = (dy - i dz)\, e^{i(t-x)}$$

and when we make the usual identifications, this can be written as

$$E = \left(0,\; e^{i(t-x)},\; -i\, e^{i(t-x)}\right).$$

Similarly $B = -iE$, so

$$B = \left(0,\; -i\, e^{i(t-x)},\; -e^{i(t-x)}\right)$$

78. ? Something funny here... Very confused...

79. $p^*(dx) = -dx$, etc. but $p^*(dt) = dt$.

So, if $F = B + E \wedge dt$ is self-dual, then

$$\therefore \; p^* F = B - E \wedge dt$$

$$\therefore \; *(p^*F) = *(B - E \wedge dt)$$

$$= -*_s E - *_s B \wedge dt$$

$$= -iB + iE \wedge dt \quad (F \text{ is self-dual})$$

$$= -i(p^*F)$$

$\therefore$ p^*F is anti-self-dual.

Similar for the case when F is anti-self-dual.