

$$59. -\frac{1}{2} \langle F, F \rangle = -\frac{1}{2} \langle B + E dt, B + E dt \rangle$$

$$= -\frac{1}{2} \left(\langle B, B \rangle + \langle E dt, E dt \rangle \right) \quad \left(\begin{array}{l} \text{there is no } dt \\ \text{term in } B \\ \text{to pair with} \\ E dt \end{array} \right)$$

$$= -\frac{1}{2} \left(\langle B, B \rangle_{\mathbb{R}^3} + \langle E, E \rangle_{\mathbb{R}^4} \langle dt, dt \rangle_{\mathbb{R}^4} \right)$$

↑
since + for
space

↑
since these are
the only nonzero pairings

$$= -\frac{1}{2} \left(\langle B, B \rangle_{\mathbb{R}^3} + \langle E, E \rangle_{\mathbb{R}^3} \cdot (-1) \right)$$

~~$$= -\frac{1}{2} \left(\langle B, B \rangle_{\mathbb{R}^3} + \langle E, E \rangle_{\mathbb{R}^3} \cdot (-1) \right)$$~~

$$= \frac{1}{2} \left(\langle E, E \rangle_{\mathbb{R}^3} - \langle B, B \rangle_{\mathbb{R}^3} \right)$$

$$= \text{Lagrangian in 58.}$$