Baez and Munian

Ex. 57 , 25/04/2010

57. We can write the inner product as

$$\langle e^1 \wedge e^2 \wedge \cdots \wedge e^p, f^1 \wedge f_2 \wedge \cdots \wedge f^p \rangle = \sum_{\text{contractions}} \langle e^1 \wedge e^2 \wedge \cdots \wedge e^p, f^{\pi(1)} \wedge \cdots f^{\pi(p)} \rangle$$

$g(e^1, f^{\pi(1)})$ etc.

So clearly

$$\langle e^{i_1} \wedge \cdots \wedge e^{i_p}, e^{j_1} \wedge \cdots \wedge e^{j_p} \rangle = 0 \quad \text{unless} \quad i_k = j_k \text{ for all } k$$

here we have $i_1 < i_2 < \cdots < i_p$

In that case we'll have

$$\langle e^{i_1} \wedge \cdots \wedge e^{i_p}, e^{i_1} \wedge \cdots \wedge e^{i_p} \rangle = 1$$

Since the only nonzero pairing is

$$e^{i_1} \wedge \cdots \wedge e^{i_p} , e^{i_1} \wedge \cdots \wedge e^{i_p} .$$