Exercise 52

Let

$$g(\cdot\,\cdot) : V \longrightarrow V^*$$

$$v \longmapsto g(v\,\cdot)$$

Suppose $g(v\,\cdot) = 0$

$\Rightarrow g(v\,\cdot)\,v = g(v,v) = 0$

Therefore $v = 0$ (nondegeneracy)

This implies that the map $g(\cdot\,\cdot)$ is one-to-one.

Using the fact that $\dim V = \dim V^*$ it follows that the map $g(\cdot\,\cdot)$ is onto, hence $g(\cdot\,\cdot)$ is an isomorphism.