I. WHAT IS FLAVOR?

The term "**flavors**" is used, in the jargon of particle physics, to describe several copies of the same gauge representation, namely several fields that are assigned the same quantum charges. Within the Standard Model, when thinking of its unbroken $SU(3)_{\rm C}\times U(1)_{\rm EM}$ gauge group, there are four different types of particles, each coming in three flavors:

- Up-type quarks in the $(3)_{+2/3}$ representation: u, c, t;
- Down-type quarks in the $(3)_{-1/3}$ representation: d, s, b;
- Charged leptons in the $(1)_{-1}$ representation: e, μ, τ;
- Neutrinos in the $(1)_0$ representation: ν_1, ν_2, ν_3.

The term "**flavor physics**" refers to interactions that distinguish between flavors. By definition, gauge interactions, namely interactions that are related to unbroken symmetries and mediated therefore by massless gauge bosons, do not distinguish among the flavors and do not constitute part of flavor physics. Within the Standard Model, flavor-physics refers to the weak and Yukawa interactions.

The term "**flavor parameters**" refers to parameters that carry flavor indices. Within the Standard Model, these are the nine masses of the charged fermions and the four "mixing parameters" (three angles and one phase) that describe the interactions of the charged weak-force carriers ($W^\pm$) with quark-antiquark pairs. If one augments the Standard Model with Majorana mass terms for the neutrinos, one should add to the list three neutrino masses and six mixing parameters (three angles and three phases) for the $W^\pm$ interactions with lepton-antilepton pairs.

The term "**flavor universal**" refers to interactions with couplings (or to parameters) that are proportional to the unit matrix in flavor space. Thus, the strong and electromagnetic interactions are flavor-universal.[1] An alternative term for "flavor-universal" is "**flavor-**

[1] In the interaction basis, the weak interactions are also flavor-universal, and one can identify the source of all flavor physics in the Yukawa interactions among the gauge-interaction eigenstates.

blind".

The term "**flavor diagonal**" refers to interactions with couplings (or to parameters) that are diagonal, but not necessarily universal, in the flavor space. Within the Standard Model, the Yukawa interactions of the Higgs particle are flavor diagonal in the mass basis.

The term "**flavor changing**" refers to processes where the initial and final flavor-numbers (that is, the number of particles of a certain flavor minus the number of anti-particles of the same flavor) are different. In "flavor changing charged current" processes, both up-type and down-type flavors, and/or both charged lepton and neutrino flavors are involved. Examples are (i) muon decay via $\mu \to e\bar{\nu}_i\nu_j$, and (ii) $K^- \to \mu^-\bar{\nu}_j$ (which corresponds, at the quark level, to $s\bar{u} \to \mu^-\bar{\nu}_j$). Within the Standard Model, these processes are mediated by the W-bosons and occur at tree level. In "**flavor changing neutral current**" (FCNC) processes, either up-type or down-type flavors but not both, and/or either charged lepton or neutrino flavors but not both, are involved. Example are (i) muon decay via $\mu \to e\gamma$ and (ii) $K_L \to \mu^+\mu^-$ (which corresponds, at the quark level, to $s\bar{d} \to \mu^+\mu^-$). Within the Standard Model, these processes do not occur at tree level, and are often highly suppressed.

Another useful term is "**flavor violation**". We will explain it later in these lectures.

II. WHY IS FLAVOR PHYSICS INTERESTING?

- Flavor physics can discover new physics or probe it before it is directly observed in experiments. Here are some examples from the past:
  - The smallness of $\frac{\Gamma(K_L\to\mu^+\mu^-)}{\Gamma(K^+\to\mu^+\nu)}$ led to predicting a fourth (the charm) quark;
  - The size of Δm_K led to a successful prediction of the charm mass;
  - The size of Δm_B led to a successful prediction of the top mass;
  - The measurement of ε_K led to predicting the third generation.
  - The measurement of neutrino flavor transitions led to the discovery of neutrino masses.
- CP violation is closely related to flavor physics. Within the Standard Model, there is a single CP violating parameter, the Kobayashi-Maskawa phase $\delta_{\rm KM}$ [1]. Baryogenesis tells us, however, that there must exist new sources of CP violation. Measurements of CP violation in flavor changing processes might provide evidence for such sources.

- The fine-tuning problem of the Higgs mass, and the puzzle of the dark matter imply that there exists new physics at, or below, the TeV scale. If such new physics had a generic flavor structure, it would contribute to flavor changing neutral current (FCNC) processes orders of magnitude above the observed rates. The question of why this does not happen constitutes the *new physics flavor puzzle*.
- Most of the charged fermion flavor parameters are small and hierarchical. The Standard Model does not provide any explanation of these features. This is the *Standard Model flavor puzzle*. The puzzle became even deeper after neutrino masses and mixings were measured because, so far, neither smallness nor hierarchy in these parameters have been established.

III. FLAVOR IN THE STANDARD MODEL

A model of elementary particles and their interactions is defined by the following ingredients: (i) The symmetries of the Lagrangian and the pattern of spontaneous symmetry breaking; (ii) The representations of fermions and scalars. The Standard Model (SM) is defined as follows:
(i) The gauge symmetry is

$$G_{\rm SM} = SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}. \tag{1}$$

It is spontaneously broken by the VEV of a single Higgs scalar, $\phi(1,2)_{1/2}$ ($\langle\phi^0\rangle = v/\sqrt{2}$):

$$G_{\rm SM} \to SU(3)_{\rm C} \times U(1)_{\rm EM}. \tag{2}$$

(ii) There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q_{Li}(3,2)_{+1/6}, \quad U_{Ri}(3,1)_{+2/3}, \quad D_{Ri}(3,1)_{-1/3}, \quad L_{Li}(1,2)_{-1/2}, \quad E_{Ri}(1,1)_{-1}. \tag{3}$$

A. The interaction basis

The Standard Model Lagrangian, $\mathcal{L}_{\rm SM}$, is the most general renormalizable Lagrangian that is consistent with the gauge symmetry (1), the particle content (3) and the pattern of spontaneous symmetry breaking (2). It can be divided to three parts:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}. \tag{4}$$

As concerns the kinetic terms, to maintain gauge invariance, one has to replace the derivative with a covariant derivative:

$$D^\mu = \partial^\mu + ig_s G_a^\mu L_a + igW_b^\mu T_b + ig' B^\mu Y. \tag{5}$$

Here G_a^μ are the eight gluon fields, W_b^μ the three weak interaction bosons and B^μ the single hypercharge boson. The L_a's are $SU(3)_{\rm C}$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_{\rm L}$ generators (the 2×2 Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and the Y's are the $U(1)_{\rm Y}$ charges. For example, for the quark doublets Q_L, we have

$$\mathcal{L}_{\rm kinetic}(Q_L) = i\overline{Q_{Li}}\gamma_\mu \left(\partial^\mu + \frac{i}{2}g_s G_a^\mu \lambda_a + \frac{i}{2}gW_b^\mu \tau_b + \frac{i}{6}g' B^\mu\right)\delta_{ij} Q_{Lj}, \tag{6}$$

while for the lepton doublets L_L^I, we have

$$\mathcal{L}_{\rm kinetic}(L_L) = i\overline{L_{Li}}\gamma_\mu \left(\partial^\mu + \frac{i}{2}gW_b^\mu \tau_b - \frac{i}{2}g' B^\mu\right)\delta_{ij} L_{Lj}. \tag{7}$$

The unit matrix in flavor space, δ_{ij}, signifies that these parts of the interaction Lagrangian are flavor-universal. In addition, they conserve CP.

The Higgs potential, which describes the scalar self interactions, is given by:

$$\mathcal{L}_{\rm Higgs} = \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger\phi)^2. \tag{8}$$

For the Standard Model scalar sector, where there is a single doublet, this part of the Lagrangian is also CP conserving.

The quark Yukawa interactions are given by

$$-\mathcal{L}_{\rm Y}^q = Y_{ij}^d \overline{Q_{Li}}\phi D_{Rj} + Y_{ij}^u \overline{Q_{Li}}\tilde{\phi} U_{Rj} + {\rm h.c.}, \tag{9}$$

(where $\tilde{\phi} = i\tau_2\phi^\dagger$) while the lepton Yukawa interactions are given by

$$-\mathcal{L}_{\rm Y}^\ell = Y_{ij}^e \overline{L_{Li}}\phi E_{Rj} + {\rm h.c.}. \tag{10}$$

This part of the Lagrangian is, in general, flavor-dependent (that is, $Y^f \not\propto \mathbf{1}$) and CP violating.

B. Global symmetries

In the absence of the Yukawa matrices Y^d, Y^u and Y^e, the SM has a large $U(3)^5$ global symmetry:

$$G_{\rm global}(Y^{u,d,e}=0) = SU(3)^3_q \times SU(3)^2_\ell \times U(1)^5, \tag{11}$$

where

$$\begin{aligned} SU(3)^3_q &= SU(3)_Q \times SU(3)_U \times SU(3)_D, \\ SU(3)^2_\ell &= SU(3)_L \times SU(3)_E, \\ U(1)^5 &= U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{\rm PQ} \times U(1)_E. \end{aligned} \tag{12}$$

Out of the five $U(1)$ charges, three can be identified with baryon number (B), lepton number (L) and hypercharge (Y), which are respected by the Yukawa interactions. The two remaining $U(1)$ groups can be identified with the PQ symmetry whereby the Higgs and D_R, E_R fields have opposite charges, and with a global rotation of E_R only.

The point that is important for our purposes is that $\mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Higgs}$ respect the non-Abelian flavor symmetry $S(3)^3_q \times SU(3)^2_\ell$, under which

$$Q_L \to V_Q Q_L, \quad U_R \to V_U U_R, \quad D_R \to V_D D_R, \quad L_L \to V_L L_L, \quad E_R \to V_E E_R, \tag{13}$$

where the V_i are unitary matrices. The Yukawa interactions (9) and (10) break the global symmetry,

$$G_{\rm global}(Y^{u,d,e} \neq 0) = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau. \tag{14}$$

(Of course, the gauged $U(1)_Y$ also remains a good symmetry.) Thus, the transformations of Eq. (13) are not a symmetry of $\mathcal{L}_{\rm SM}$. Instead, they correspond to a change of the interaction basis. These observations also offer an alternative way of defining flavor physics: it refers to interactions that break the $SU(3)^5$ symmetry (13). Thus, the term "**flavor violation**" is often used to describe processes or parameters that break the symmetry.

One can think of the quark Yukawa couplings as spurions that break the global $SU(3)^3_q$ symmetry (but are neutral under $U(1)_B$),

$$Y^u \sim (3, \bar{3}, 1)_{SU(3)^3_q}, \quad Y^d \sim (3, 1, \bar{3})_{SU(3)^3_q}, \tag{15}$$

and of the lepton Yukawa couplings as spurions that break the global $SU(3)^2_\ell$ symmetry (but are neutral under $U(1)_e \times U(1)_\mu \times U(1)_\tau$),

$$Y^e \sim (3, \bar{3})_{SU(3)^2_\ell}. \tag{16}$$

The spurion formalism is convenient for several purposes: parameter counting (see below), identification of flavor suppression factors (see Section V), and the idea of minimal flavor violation (which is beyond the scope of this course).

C. Counting parameters

How many independent parameters are there in $\mathcal{L}_{\rm Y}^q$? The two Yukawa matrices, Y^u and Y^d, are 3×3 and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. The pattern of $G_{\rm global}$ breaking means that there is freedom to remove 9 real and 17 imaginary parameters (the number of parameters in three 3×3 unitary matrices minus the phase related to $U(1)_B$). For example, we can use the unitary transformations $Q_L\to V_Q Q_L$, $U_R\to V_U U_R$ and $D_R\to V_D D_R$, to lead to the following interaction basis:

$$Y^d=\lambda_d,\quad Y^u=V^\dagger\lambda_u, \tag{17}$$

where $\lambda_{d,u}$ are diagonal,

$$\lambda_d={\rm diag}(y_d,y_s,y_b),\quad \lambda_u={\rm diag}(y_u,y_c,y_t), \tag{18}$$

while V is a unitary matrix that depends on three real angles and one complex phase. We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. In the mass basis, we will identify the nine real parameters as six quark masses and three mixing angles, while the single phase is $\delta_{\rm KM}$.

How many independent parameters are there in $\mathcal{L}_{\rm Y}^\ell$? The Yukawa matrix Y^e is 3×3 and complex. Consequently, there are 9 real and 9 imaginary parameters in this matrix. There is, however, freedom to remove 6 real and 9 imaginary parameters (the number of parameters in two 3×3 unitary matrices minus the phases related to $U(1)^3$). For example, we can use the unitary transformations $L_L\to V_L L_L$ and $E_R\to V_E E_R$, to lead to the following interaction basis:

$$Y^e=\lambda_e={\rm diag}(y_e,y_\mu,y_\tau). \tag{19}$$

We conclude that there are 3 real lepton flavor parameters. In the mass basis, we will identify these parameters as the three charged lepton masses. We must, however, modify the model when we take into account the evidence for neutrino masses.

D. The mass basis

Upon the replacement $\mathcal{R}e(\phi^0) \to \frac{v+h^0}{\sqrt{2}}$, the Yukawa interactions (9) give rise to the mass matrices

$$M_q = \frac{v}{\sqrt{2}} Y^q. \tag{20}$$

The mass basis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices V_{qL} and V_{qR} such that

$$V_{qL} M_q V_{qR}^\dagger = M_q^{\rm diag} \equiv \frac{v}{\sqrt{2}} \lambda_q. \tag{21}$$

The four matrices V_{dL}, V_{dR}, V_{uL} and V_{uR} are then the ones required to transform to the mass basis. For example, if we start from the special basis (17), we have $V_{dL} = V_{dR} = V_{uR} = \mathbf{1}$ and $V_{uL} = V$. The combination $V_{uL} V_{dL}^\dagger$ is independent of the interaction basis from which we start this procedure.

We denote the left-handed quark mass eigenstates as U_L and D_L. The charged current interactions for quarks [that is the interactions of the charged $SU(2)_{\rm L}$ gauge bosons $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$], which in the interaction basis are described by (6), have a complicated form in the mass basis:

$$-\mathcal{L}_{W^\pm}^q = \frac{g}{\sqrt{2}} \overline{U_{Li}} \gamma^\mu V_{ij} D_{Lj} W_\mu^+ + {\rm h.c.}. \tag{22}$$

where V is the 3×3 unitary matrix ($VV^\dagger = V^\dagger V = \mathbf{1}$) that appeared in Eq. (17). For a general interaction basis,

$$V = V_{uL} V_{dL}^\dagger. \tag{23}$$

V is the Cabibbo-Kobayashi-Maskawa (CKM) *mixing matrix* for quarks [1, 2]. As a result of the fact that V is not diagonal, the $W^\pm$ gauge bosons couple to quark mass eigenstates of different generations. Within the Standard Model, this is the only source of *flavor changing* quark interactions.

The detailed structure of the CKM matrix, its parametrization, and the constraints on its elements are described in Appendix A.

IV. TESTING THE CKM MECHANISM

Measurements of rates, mixing, and CP asymmetries in B decays in the two B factories, BaBar abd Belle, and in the two Tevatron detectors, CDF and D0, signified a new era in our understanding of CP violation. The progress is both qualitative and quantitative. Various basic questions concerning CP and flavor violation have received, for the first time, answers based on experimental information. These questions include, for example,

- Is the Kobayashi-Maskawa mechanism at work (namely, is $\delta_{\rm KM} \neq 0$)?
- Does the KM phase dominate the observed CP violation?

As a first step, one may assume the SM and test the overall consistency of the various measurements. However, the richness of data from the B factories allow us to go a step further and answer these questions model independently, namely allowing new physics to contribute to the relevant processes. We here explain the way in which this analysis proceeds.

A. $S_{\psi K_S}$

The CP asymmetry in $B \to \psi K_S$ decays plays a major role in testing the KM mechanism. Before we explain the test itself, we should understand why the theoretical interpretation of the asymmetry is exceptionally clean, and what are the theoretical parameters on which it depends, within and beyond the Standard Model.

The CP asymmetry in neutral meson decays into final CP eigenstates f_{CP} is defined as follows:

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}] - d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}]}{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}] + d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}]} . \tag{24}$$

A detailed evaluation of this asymmetry is given in Appendix B. It leads to the following form:

$$\begin{aligned} \mathcal{A}_{f_{CP}}(t) &= S_{f_{CP}} \sin(\Delta m t) - C_{f_{CP}} \cos(\Delta m t), \\ S_{f_{CP}} &\equiv \frac{2\,\mathcal{I}m(\lambda_{f_{CP}})}{1+|\lambda_{f_{CP}}|^2}, \quad C_{f_{CP}} \equiv \frac{1-|\lambda_{f_{CP}}|^2}{1+|\lambda_{f_{CP}}|^2} , \end{aligned} \tag{25}$$

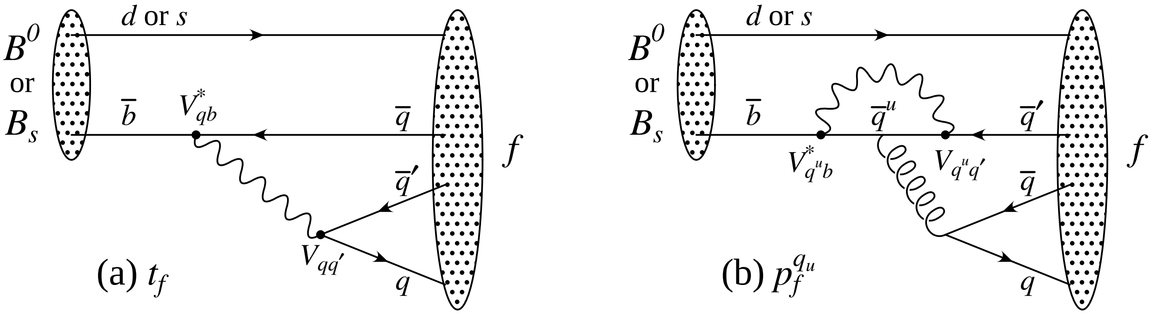
where

$$\lambda_{f_{CP}} = e^{-i\phi_B}(\overline{A}_{f_{CP}}/A_{f_{CP}}) . \tag{26}$$

Here ϕ_B refers to the phase of M_{12} [see Eq. (B23)]. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_B} = (V_{tb}^* V_{td})/(V_{tb} V_{td}^*) . \tag{27}$$

FIG. 1: Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s \to f$ via a $\bar{b} \to \bar{q}q\bar{q}'$ quark-level process.



The decay amplitudes A_f and $\overline{A}_f$ are defined in Eq. (B1).

The $B^0 \to J/\psi K^0$ decay [3, 4] proceeds via the quark transition $\bar{b} \to \bar{c}c\bar{s}$. There are contributions from both tree (t) and penguin (p^{q_u}, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 1) which carry different weak phases:

$$A_f = (V^*_{cb} V_{cs})\, t_f + \sum_{q_u=u,c,t} \left(V^*_{q_u b} V_{q_u s}\right) p_f^{q_u} \; . \tag{28}$$

(The distinction between tree and penguin contributions is a heuristic one, the separation by the operator that enters is more precise. For a detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, see, for example, ref. [5].) Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations:

$$A_{\psi K} = (V^*_{cb} V_{cs})\, T_{\psi K} + (V^*_{ub} V_{us})\, P^u_{\psi K}, \tag{29}$$

where $T_{\psi K} = t_{\psi K} + p^c_{\psi K} - p^t_{\psi K}$ and $P^u_{\psi K} = p^u_{\psi K} - p^t_{\psi K}$. A subtlety arises in this decay that is related to the fact that $B^0 \to J/\psi K^0$ and $\overline{B}^0 \to J/\psi \overline{K}^0$. A common final state, e.g. $J/\psi K_S$, can be reached via $K^0 - \overline{K}^0$ mixing. Consequently, the phase factor corresponding to neutral K mixing, $e^{-i\phi_K} = (V^*_{cd} V_{cs})/(V_{cd} V^*_{cs})$, plays a role:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{(V_{cb} V^*_{cs})\, T_{\psi K} + (V_{ub} V^*_{us})\, P^u_{\psi K}}{(V^*_{cb} V_{cs})\, T_{\psi K} + (V^*_{ub} V_{us})\, P^u_{\psi K}} \times \frac{V^*_{cd} V_{cs}}{V_{cd} V^*_{cs}}. \tag{30}$$

The crucial point is that, for $B \to J/\psi K_S$ and other $\bar{b} \to \bar{c}c\bar{s}$ processes, we can neglect the P^u contribution to $A_{\psi K}$, in the SM, to an approximation that is better than one percent:

$$|P^u_{\psi K}/T_{\psi K}| \times |V_{ub}/V_{cb}| \times |V_{us}/V_{cs}| \sim (\text{loop factor}) \times 0.1 \times 0.23 \lesssim 0.005. \tag{31}$$

Thus, to an accuracy better than one percent,

$$\lambda_{\psi K_S} = \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right) \left(\frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}\right) = -e^{-2i\beta}, \tag{32}$$

where β is defined in Eq. (A9), and consequently

$$S_{\psi K_S} = \sin 2\beta, \quad C_{\psi K_S} = 0 \ . \tag{33}$$

(Below the percent level, several effects modify this equation [6–9].)

When we consider extensions of the SM, we still do not expect any significant new contribution to the tree level decay, $b \to c\bar{c}s$, beyond the SM W-mediated diagram. Thus, the expression $\bar{A}_{\psi K_S}/A_{\psi K_S} = (V_{cb}V_{cd}^*)/(V_{cb}^*V_{cd})$ remains valid, though the approximation of neglecting sub-dominant phases can be somewhat less accurate than Eq. (31). On the other hand, M_{12}, the $B^0 - \overline{B}^0$ mixing amplitude, can in principle get large and even dominant contributions from new physics. We can parameterize the modification to the SM in terms of two parameters, r_d^2 signifying the change in magnitude, and $2\theta_d$ signifying the change in phase:

$$M_{12} = r_d^2 \ e^{2i\theta_d} \ M_{12}^{\rm SM}(\rho, \eta). \tag{34}$$

This leads to the following generalization of Eq. (33):

$$S_{\psi K_S} = \sin(2\beta + 2\theta_d), \quad C_{\psi K_S} = 0 \ . \tag{35}$$

The experimental measurements give the following ranges [10]:

$$S_{\psi K_S} = +0.68 \pm 0.02, \quad C_{\psi K_S} = +0.005 \pm 0.017 \ . \tag{36}$$

B. Self-consistency of the CKM assumption

The three generation standard model has room for CP violation, through the KM phase in the quark mixing matrix. Yet, one would like to make sure that indeed CP is violated by the SM interactions, namely that $\sin\delta_{\rm KM} \neq 0$. If we establish that this is the case, we would further like to know whether the SM contributions to CP violating observables are dominant. More quantitatively, we would like to put an upper bound on the ratio between the new physics and the SM contributions.

As a first step, one can assume that flavor changing processes are fully described by the SM, and check the consistency of the various measurements with this assumption. There are four relevant mixing parameters, which can be taken to be the Wolfenstein parameters λ, A, ρ and η defined in Eq. (A4). The values of λ and A are known rather accurately [11] from, respectively, $K \to \pi\ell\nu$ and $b \to c\ell\nu$ decays:

$$\lambda = 0.2254 \pm 0.0007, \quad A = 0.811^{+0.022}_{-0.012}. \tag{37}$$

Then, one can express all the relevant observables as a function of the two remaining parameters, ρ and η, and check whether there is a range in the $\rho - \eta$ plane that is consistent with all measurements. The list of observables includes the following:

- The rates of inclusive and exclusive charmless semileptonic B decays depend on $|V_{ub}|^2 \propto \rho^2 + \eta^2$;
- The CP asymmetry in $B \to \psi K_S$, $S_{\psi K_S} = \sin 2\beta = \frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2}$;
- The rates of various $B \to DK$ decays depend on the phase γ, where $e^{i\gamma} = \frac{\rho+i\eta}{\sqrt{\rho^2+\eta^2}}$;
- The rates of various $B \to \pi\pi, \rho\pi, \rho\rho$ decays depend on the phase $\alpha = \pi - \beta - \gamma$;
- The ratio between the mass splittings in the neutral B and B_s systems is sensitive to $|V_{td}/V_{ts}|^2 = \lambda^2[(1-\rho)^2 + \eta^2]$;
- The CP violation in $K \to \pi\pi$ decays, ϵ_K, depends in a complicated way on ρ and η.

The resulting constraints are shown in Fig. 2.

The consistency of the various constraints is impressive. In particular, the following ranges for ρ and η can account for all the measurements [11]:

$$\rho = +0.131^{+0.026}_{-0.013}, \quad \eta = +0.345 \pm 0.014. \tag{38}$$

One can make then the following statement [13]:

Very likely, CP violation in flavor changing processes is dominated by the Kobayashi-Maskawa phase.

In the next two subsections, we explain how we can remove the phrase "very likely" from this statement, and how we can quantify the KM-dominance.

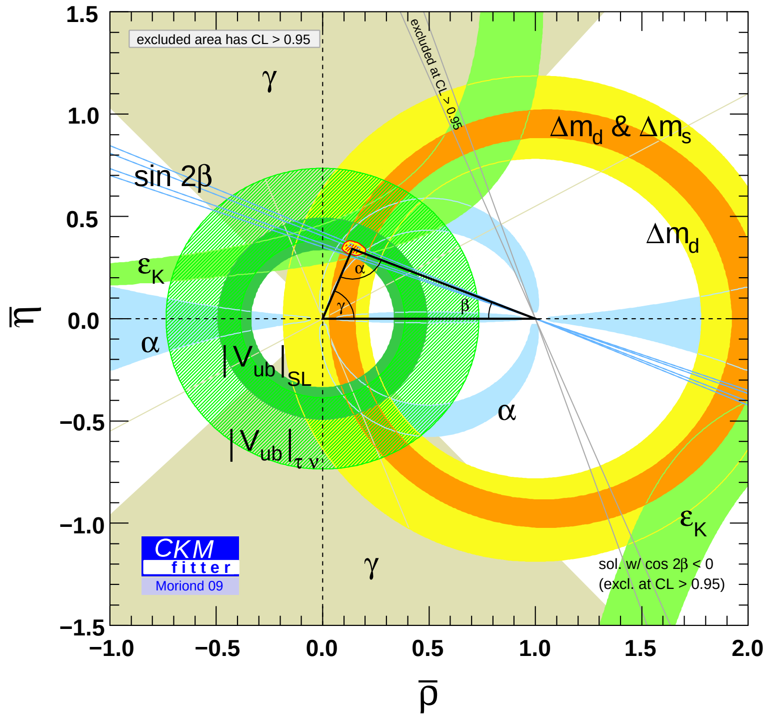


FIG. 2: Allowed region in the ρ, η plane. Superimposed are the individual constraints from charmless semileptonic B decays ($|V_{ub}/V_{cb}|$), mass differences in the B^0 (Δm_d) and B_s (Δm_s) neutral meson systems, and CP violation in $K \to \pi\pi$ (ε_K), $B \to \psi K$ ($\sin 2\beta$), $B \to \pi\pi, \rho\pi, \rho\rho$ (α), and $B \to DK$ (γ). Taken from [12].

C. Is the KM mechanism at work?

In proving that the KM mechanism is at work, we assume that charged-current tree-level processes are dominated by the W-mediated SM diagrams (see, for example, [14]). This is a very plausible assumption. I am not aware of any viable well-motivated model where this assumption is not valid. Thus we can use all tree level processes and fit them to ρ and η, as we did before. The list of such processes includes the following:

1. Charmless semileptonic B-decays, $b \to u\ell\nu$, measure R_u [see Eq. (A8)].

2. $B \to DK$ decays, which go through the quark transitions $b \to c\bar{u}s$ and $b \to u\bar{c}s$, measure the angle γ [see Eq. (A9)].

3. $B \to \rho\rho$ decays (and, similarly, $B \to \pi\pi$ and $B \to \rho\pi$ decays) go through the quark

transition $b \to u\bar{u}d$. With an isospin analysis, one can determine the relative phase between the tree decay amplitude and the mixing amplitude. By incorporating the measurement of $S_{\psi K_S}$, one can subtract the phase from the mixing amplitude, finally providing a measurement of the angle γ [see Eq. (A9)].

In addition, we can use loop processes, but then we must allow for new physics contributions, in addition to the (ρ, η)-dependent SM contributions. Of course, if each such measurement adds a separate mode-dependent parameter, then we do not gain anything by using this information. However, there is a number of observables where the only relevant loop process is $B^0 - \overline{B}^0$ mixing. The list includes $S_{\psi K_S}$, Δm_B and the CP asymmetry in semileptonic B decays:

$$\begin{aligned} S_{\psi K_S} &= \sin(2\beta + 2\theta_d), \\ \Delta m_B &= r_d^2 (\Delta m_B)^{\rm SM}, \\ \mathcal{A}_{\rm SL} &= -\mathcal{R}e\left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\sin 2\theta_d}{r_d^2} + \mathcal{I}m\left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\cos 2\theta_d}{r_d^2}. \end{aligned} \tag{39}$$

As explained above, such processes involve two new parameters [see Eq. (34)]. Since there are three relevant observables, we can further tighten the constraints in the (ρ, η)-plane. Similarly, one can use measurements related to $B_s - \overline{B}_s$ mixing. One gains three new observables at the cost of two new parameters (see, for example, [15]).

The results of such fit, projected on the $\rho - \eta$ plane, can be seen in Fig. 3. It gives [12]

$$\eta = 0.44^{+0.05}_{-0.23} \quad (3\sigma). \tag{40}$$

[A similar analysis in Ref. [16] obtains the 3σ range $(0.31 - 0.46)$.] It is clear that $\eta \neq 0$ is well established:

The Kobayashi-Maskawa mechanism of CP violation is at work.

Another way to establish that CP is violated by the CKM matrix is to find, within the same procedure, the allowed range for $\sin 2\beta$ [16]:

$$\sin 2\beta^{\rm tree} = 0.80 \pm 0.03. \tag{41}$$

Thus, $\beta \neq 0$ is well established.

The consistency of the experimental results (36) with the SM predictions (33,41) means that the KM mechanism of CP violation dominates the observed CP violation. In the next subsection, we make this statement more quantitative.

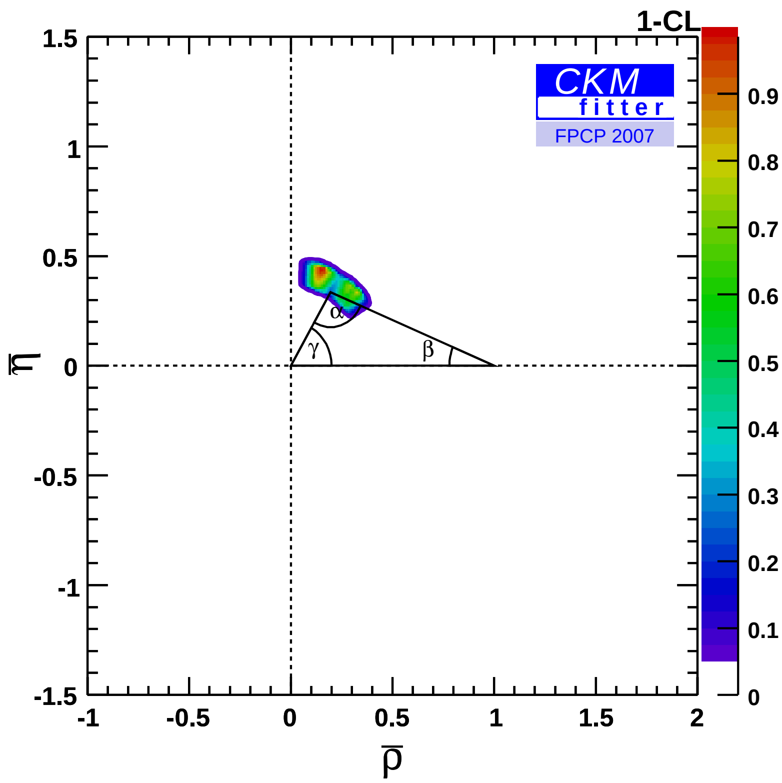


FIG. 3: The allowed region in the $\rho - \eta$ plane, assuming that tree diagrams are dominated by the Standard Model [12].

D. How much can new physics contribute to $B^0 - \overline{B}^0$ mixing?

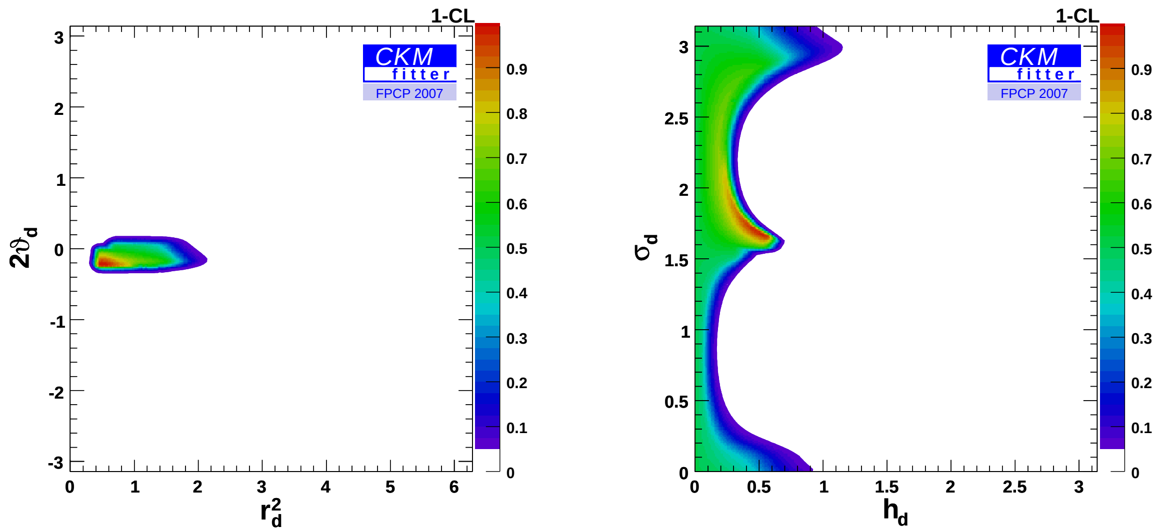
All that we need to do in order to establish whether the SM dominates the observed CP violation, and to put an upper bound on the new physics contribution to $B^0 - \overline{B}^0$ mixing, is to project the results of the fit performed in the previous subsection on the $r_d^2 - 2\theta_d$ plane. If we find that $\theta_d \ll \beta$, then the SM dominance in the observed CP violation will be established. The constraints are shown in Fig. 4(a). Indeed, $\theta_d \ll \beta$.

An alternative way to present the data is to use the h_d, σ_d parametrization,

$$r_d^2 e^{2i\theta_d} = 1 + h_d e^{2i\sigma_d}. \tag{42}$$

While the r_d, θ_d parameters give the relation between the full mixing amplitude and the SM one, and are convenient to apply to the measurements, the h_d, σ_d parameters give the relation between the new physics and SM contributions, and are more convenient in testing

FIG. 4: Constraints in the (a) $r_d^2 - 2\theta_d$ plane, and (b) $h_d - \sigma_d$ plane, assuming that NP contributions to tree level processes are negligible [12].



theoretical models:

$$h_d e^{2i\sigma_d} = \frac{M_{12}^{\rm NP}}{M_{12}^{\rm SM}}. \tag{43}$$

The constraints in the $h_d - \sigma_d$ plane are shown in Fig. 4(b). We can make the following two statements:

1. A new physics contribution to $B^0 - \overline{B}^0$ mixing amplitude that carries a phase that is significantly different from the KM phase is constrained to lie below the 20-30% level.

2. A new physics contribution to the $B^0 - \overline{B}^0$ mixing amplitude which is aligned with the KM phase is constrained to be at most comparable to the CKM contribution.

One can reformulate these statements as follows:

1. The KM mechanism dominates CP violation in $B^0 - \overline{B}^0$ mixing.

2. The CKM mechanism is a major player in $B^0 - \overline{B}^0$ mixing.

V. THE NEW PHYSICS FLAVOR PUZZLE

Given that the SM is only an effective low energy theory, non-renormalizable terms must be added to $\mathcal{L}_{\rm SM}$ of Eq. (4). These are terms of dimension higher than four in the fields

TABLE I: Measurements related to neutral meson mixing

Sector	CP-conserving	CP-violating
sd	$\Delta m_K/m_K = 7.0\times 10^{-15}$	$\epsilon_K = 2.3\times 10^{-3}$
cu	$\Delta m_D/m_D = 8.7\times 10^{-15}$	$A_\Gamma/y_{\rm CP} \lesssim 0.2$
bd	$\Delta m_B/m_B = 6.3\times 10^{-14}$	$S_{\psi K} = +0.67\pm 0.02$
bs	$\Delta m_{B_s}/m_{B_s} = 2.1\times 10^{-12}$	$S_{\psi\phi} = -0.04\pm 0.09$

which, therefore, have couplings that are inversely proportional to the scale of new physics $\Lambda_{\rm NP}$. As concerns quark flavor physics, consider, for example, the following dimension-six, four-fermion, flavor changing operators:

$$\mathcal{L}_{\Delta F=2} = \frac{z_{sd}}{\Lambda_{\rm NP}^2}(\overline{d_L}\gamma_\mu s_L)^2 + \frac{z_{cu}}{\Lambda_{\rm NP}^2}(\overline{c_L}\gamma_\mu u_L)^2 + \frac{z_{bd}}{\Lambda_{\rm NP}^2}(\overline{d_L}\gamma_\mu b_L)^2 + \frac{z_{bs}}{\Lambda_{\rm NP}^2}(\overline{s_L}\gamma_\mu b_L)^2. \tag{44}$$

Each of these terms contributes to the mass splitting between the corresponding two neutral mesons. For example, the term $\mathcal{L}_{\Delta B=2}\propto(\overline{d_L}\gamma_\mu b_L)^2$ contributes to Δm_B, the mass difference between the two neutral B-mesons. We use $M_{12}^B = \frac{1}{2m_B}\langle B^0|\mathcal{L}_{\Delta F=2}|\overline{B}^0\rangle$ and

$$\langle B^0|(\overline{d_{La}}\gamma^\mu b_{La})(\overline{d_{Lb}}\gamma_\mu b_{Lb})|\overline{B}^0\rangle = -\frac{1}{3}m_B^2 f_B^2 B_B. \tag{45}$$

Analogous expressions hold for the other neutral mesons.[2] This leads to $\Delta m_B/m_B = 2|M_{12}^B|/m_B \sim (|z_{bd}|/3)(f_B/\Lambda_{\rm NP})^2$.

The experimental results for CP conserving and CP violating observables related to neutral meson mixing (mass splittings and CP asymmetries in tree level decays, respectively) are given in Table I.

The measurements quoted in Table I lead, for a given value of $|z_{ij}|$ and $z_{ij}^I\equiv\mathcal{I}m(z_{ij})$, to lower bounds on the scale $\Lambda_{\rm NP}$. In Table II we give the bounds that correspond to $|z_{ij}|=1$ and to $z_{ij}^I=1$. The bounds scale like $\sqrt{z_{ij}}$ and $\sqrt{z_{ij}^I}$, respectively.

If the new physics has a generic flavor structure, that is $z_{ij}=\mathcal{O}(1)$, then its scale must be above 10^3-10^4 TeV (or, if the leading contributions involve electroweak loops, above

[2] The PDG [11] quotes the following values, extracted from leptonic charged meson decays: $f_K\approx 0.16\ GeV$, $f_D\approx 0.23\ GeV$, $f_B\approx 0.18\ GeV$. We further use $f_{B_s}\approx 0.20\ GeV$.

TABLE II: Lower bounds on the scale of new physics $\Lambda_{\rm NP}$, in units of TeV. The bounds from CP conserving (violating) observables scale like $\sqrt{z_{ij}}$ ($\sqrt{z^I_{ij}}$).

ij	CP-conserving	CP-violating
sd	1×10^3	2×10^4
cu	1×10^3	3×10^3
bd	4×10^2	8×10^2
bs	7×10^1	2×10^2

10^2-10^3 TeV).[3] *If indeed $\Lambda_{\rm NP}\gg TeV$, it means that we have misinterpreted the hints from the fine-tuning problem and the dark matter puzzle.*

There is, however, another way to look at these constraints:

$$\begin{aligned} z_{sd} &\lesssim 8\times10^{-7}\ (\Lambda_{\rm NP}/TeV)^2,\\ z_{cu} &\lesssim 5\times10^{-7}\ (\Lambda_{\rm NP}/TeV)^2,\\ z_{bd} &\lesssim 5\times10^{-6}\ (\Lambda_{\rm NP}/TeV)^2,\\ z_{bs} &\lesssim 2\times10^{-4}\ (\Lambda_{\rm NP}/TeV)^2, \end{aligned} \tag{46}$$

$$\begin{aligned} z^I_{sd} &\lesssim 6\times10^{-9}\ (\Lambda_{\rm NP}/TeV)^2,\\ z^I_{cu} &\lesssim 1\times10^{-7}\ (\Lambda_{\rm NP}/TeV)^2,\\ z^I_{bd} &\lesssim 1\times10^{-6}\ (\Lambda_{\rm NP}/TeV)^2,\\ z^I_{bs} &\lesssim 2\times10^{-5}\ (\Lambda_{\rm NP}/TeV)^2. \end{aligned} \tag{47}$$

It could be that the scale of new physics is of order TeV, but its flavor structure is far from generic. .Specifically, if new particles at the TeV scale couple to the SM fermions, then there are two ways in which their contributions to FCNC processes, such as neutral meson mixing, can be suppressed: degeneracy and alignment. Either of these principles, or a combination of both, signifies non-generic structure.

[3] The bounds from the corresponding four-fermi terms with LR structure, instead of the LL structure of Eq. (44), are even stronger.

One can use the language of effective operators also for the SM, integrating out all particles significantly heavier than the neutral mesons (that is, the top, the Higgs and the weak gauge bosons). Thus, the scale is $\Lambda_{\rm SM} \sim m_W$. Since the leading contributions to neutral meson mixings come from box diagrams, the z_{ij} coefficients are suppressed by α_2^2. To identify the relevant flavor suppression factor, one can employ the spurion formalism. For example, the flavor transition that is relevant to $B^0 - \overline{B}^0$ mixing involves $\overline{d_L} b_L$ which transforms as $(8,1,1)_{SU(3)_q^3}$. The leading contribution must then be proportional to $(Y^u Y^{u\dagger})_{13} \propto y_t^2 V_{tb} V_{td}^*$. Indeed, an explicit calculation (using VIA for the matrix element and neglecting QCD corrections) gives[4]

$$\frac{2M_{12}^B}{m_B} \approx -\frac{\alpha_2^2}{12}\frac{f_B^2}{m_W^2} S_0(x_t)(V_{tb}V_{td}^*)^2, \tag{48}$$

where $x_i = m_i^2/m_W^2$ and

$$S_0(x) = \frac{x}{(1-x)^2}\left[1 - \frac{11x}{4} + \frac{x^2}{4} - \frac{3x^2 \ln x}{2(1-x)}\right]. \tag{49}$$

Similar spurion analyses, or explicit calculations, allow us to extract the weak and flavor suppression factors that apply in the SM:

$$\begin{aligned}
\mathcal{I}m(z_{sd}^{\rm SM}) &\sim \alpha_2^2 y_t^2 |V_{td}V_{ts}|^2 \sim 1 \times 10^{-10}, \\
z_{sd}^{\rm SM} &\sim \alpha_2^2 y_c^2 |V_{cd}V_{cs}|^2 \sim 5 \times 10^{-9}, \\
\mathcal{I}m(z_{cu}^{\rm SM}) &\sim \alpha_2^2 y_b^2 |V_{ub}V_{cb}|^2 \sim 2 \times 10^{-14}, \\
z_{bd}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{td}V_{tb}|^2 \sim 7 \times 10^{-8}, \\
z_{bs}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{ts}V_{tb}|^2 \sim 2 \times 10^{-6}.
\end{aligned} \tag{50}$$

(We did not include $z_{cu}^{\rm SM}$ in the list because it requires a more detailed consideration. The naively leading short distance contribution is $\propto \alpha_2^2 (y_s^4/y_c^2)|V_{cs}V_{us}|^2 \sim 5 \times 10^{-13}$. However, higher dimension terms can replace a y_s^2 factor with $(\Lambda/m_D)^2$ [18]. Moreover, long distance contributions are expected to dominate. In particular, peculiar phase space effects [19, 20] have been identified which are expected to enhance Δm_D to within an order of magnitude of the its measured value. The CP violating part, on the other hand, is dominated by short distance physics.)

[4] A detailed derivation can be found in Appendix B of [17].

It is clear then that contributions from new physics at $\Lambda_{\rm NP} \sim 1\ TeV$ should be suppressed by factors that are comparable or smaller than the SM ones. Why does that happen? This is the new physics flavor puzzle.

The fact that the flavor structure of new physics at the TeV scale must be non-generic means that flavor measurements are a good probe of the new physics. Perhaps the best-studied example is that of supersymmetry. Here, the spectrum of the superpartners and the structure of their couplings to the SM fermions will allow us to probe the mechanism of dynamical supersymmetry breaking.

VI. CONCLUSIONS

(i) Measurements of CP violating B-meson decays have established that the Kobayashi-Maskawa mechanism is the dominant source of the observed CP violation.

(ii) Measurements of flavor changing B-meson decays have established the the Cabibbo-Kobayashi-Maskawa mechanism is a major player in flavor violation.

(iii) The consistency of all these measurements with the CKM predictions sharpens the new physics flavor puzzle: If there is new physics at, or below, the TeV scale, then its flavor structure must be highly non-generic.

The huge progress in flavor physics in recent years has provided answers to many questions. At the same time, new questions arise. The LHC era is likely to provide more answers and more questions.

APPENDIX A: THE CKM MATRIX

The CKM matrix V is a 3×3 unitary matrix. Its form, however, is not unique:

(i) There is freedom in defining V in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, *i.e.* $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. The elements of V are written as follows:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{A1}$$

(ii) There is further freedom in the phase structure of V. This means that the number of physical parameters in V is smaller than the number of parameters in a general unitary

3×3 matrix which is nine (three real angles and six phases). Let us define P_q $(q = u, d)$ to be diagonal unitary (phase) matrices. Then, if instead of using V_{qL} and V_{qR} for the rotation (21) to the mass basis we use $\tilde{V}_{qL}$ and $\tilde{V}_{qR}$, defined by $\tilde{V}_{qL} = P_q V_{qL}$ and $\tilde{V}_{qR} = P_q V_{qR}$, we still maintain a legitimate mass basis since $M_q^{\rm diag}$ remains unchanged by such transformations. However, V does change:

$$V \to P_u V P_d^*. \tag{A2}$$

This freedom is fixed by demanding that V has the minimal number of phases. In the three generation case V has a single phase. (There are five phase differences between the elements of P_u and P_d and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase $\delta_{\rm KM}$ which is the single source of CP violation in the quark sector of the Standard Model [1].

The fact that V is unitary and depends on only four independent physical parameters can be made manifest by choosing a specific parametrization. The standard choice is [21]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{A3}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The θ_{ij}'s are the three real mixing parameters while δ is the Kobayashi-Maskawa phase. It is known experimentally that $s_{13} \ll s_{23} \ll s_{12} \ll 1$. It is convenient to choose an approximate expression where this hierarchy is manifest. This is the Wolfenstein parametrization, where the four mixing parameters are (λ, A, ρ, η) with $\lambda = |V_{us}| = 0.23$ playing the role of an expansion parameter and η representing the CP violating phase [22, 23]:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}. \tag{A4}$$

A very useful concept is that of the *unitarity triangles*. The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \tag{A5}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \tag{A6}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{A7}$$

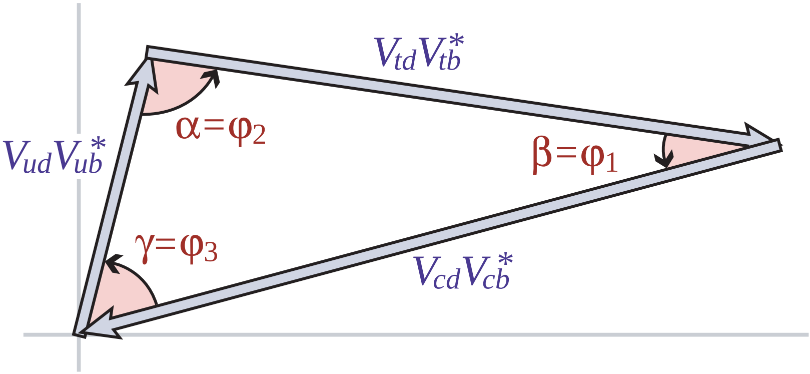


FIG. 5: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (A7) only. The unitarity triangle related to Eq. (A7) is depicted in Fig. 5.

The rescaled unitarity triangle is derived from (A7) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ, η). The area of the rescaled unitarity triangle is $|\eta|/2$.

Depicting the rescaled unitarity triangle in the (ρ, η) plane, the lengths of the two complex sides are

$$R_u \equiv \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| = \sqrt{\rho^2+\eta^2}, \quad R_t \equiv \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right| = \sqrt{(1-\rho)^2+\eta^2}. \tag{A8}$$

The three angles of the unitarity triangle are defined as follows [24, 25]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{A9}$$

They are physical quantities and can be independently measured by CP asymmetries in B decays. It is also useful to define the two small angles of the unitarity triangles (A6,A5):

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{A10}$$

The λ and A parameters are very well determined at present, see Eq. (37). The main effort in CKM measurements is thus aimed at improving our knowledge of ρ and η:

$$\rho = 0.131^{+0.026}_{-0.013}, \quad \eta = 0.345 \pm 0.013. \tag{A11}$$

The present status of our knowledge is best seen in a plot of the various constraints and the final allowed region in the $\rho - \eta$ plane. This is shown in Fig. 2.

APPENDIX B: CPV IN B DECAYS TO FINAL CP EIGENSTATES

We define decay amplitudes of B (which could be charged or neutral) and its CP conjugate $\overline{B}$ to a multi-particle final state f and its CP conjugate $\overline{f}$ as

$$A_f = \langle f|\mathcal{H}|B\rangle \quad , \quad \overline{A}_f = \langle f|\mathcal{H}|\overline{B}\rangle \quad , \quad A_{\overline{f}} = \langle \overline{f}|\mathcal{H}|B\rangle \quad , \quad \overline{A}_{\overline{f}} = \langle \overline{f}|\mathcal{H}|\overline{B}\rangle \ , \tag{B1}$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of CP on these states introduces phases ξ_B and ξ_f according to

$$\begin{aligned} CP\ |B\rangle \ &= \ e^{+i\xi_B}\,|\overline{B}\rangle \quad , \quad CP\ |f\rangle = e^{+i\xi_f}\,|\overline{f}\rangle \ , \\ CP\ |\overline{B}\rangle \ &= \ e^{-i\xi_B}\,|B\rangle \quad , \quad CP\ |\overline{f}\rangle = e^{-i\xi_f}\,|f\rangle \ , \end{aligned} \tag{B2}$$

so that $(CP\ )^2 = 1$. The phases ξ_B and ξ_f are arbitrary and unphysical because of the flavor symmetry of the strong interaction. If CP is conserved by the dynamics, $[CP\ , \mathcal{H}] = 0$, then A_f and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_B)}\,A_f \ . \tag{B3}$$

A state that is initially a superposition of B^0 and $\overline{B}^0$, say

$$|\psi(0)\rangle = a(0)|B^0\rangle + b(0)|\overline{B}^0\rangle \ , \tag{B4}$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\overline{B}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots \ . \tag{B5}$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times t in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [26]. The simplified time evolution is determined by a 2×2 effective Hamiltonian $\mathcal{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathcal{H}$, can be written in terms of Hermitian matrices M and Γ as

$$\mathcal{H} = M - \frac{i}{2}\Gamma\,. \tag{B6}$$

M and Γ are associated with $(B^0,\overline{B}^0) \leftrightarrow (B^0,\overline{B}^0)$ transitions via off-shell (dispersive) and on-shell (absorptive) intermediate states, respectively. Diagonal elements of M and Γ are associated with the flavor-conserving transitions $B^0 \to B^0$ and $\overline{B}^0 \to \overline{B}^0$ while off-diagonal elements are associated with flavor-changing transitions $B^0 \leftrightarrow \overline{B}^0$.

The eigenvectors of $\mathcal{H}$ have well defined masses and decay widths. We introduce complex parameters p and q to specify the components of the strong interaction eigenstates, B^0 and $\overline{B}^0$, in the light (B_L) and heavy (B_H) mass eigenstates:

$$|B_{L,H}\rangle = p|B^0\rangle \pm q|\overline{B}^0\rangle \tag{B7}$$

with the normalization $|p|^2+|q|^2=1$. The special form of Eq. (B7) is related to the fact that CPT imposes $M_{11}=M_{22}$ and $\Gamma_{11}=\Gamma_{22}$. Solving the eigenvalue problem gives

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}}\,. \tag{B8}$$

If either CP or T is a symmetry of $\mathcal{H}$, then M_{12} and Γ_{12} are relatively real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_B} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1\,, \tag{B9}$$

where ξ_B is the arbitrary unphysical phase introduced in Eq. (B2).

The real and imaginary parts of the eigenvalues of $\mathcal{H}$ corresponding to $|B_{L,H}\rangle$ represent their masses and decay-widths, respectively. The mass difference Δm_B and the width difference $\Delta\Gamma_B$ are defined as follows:

$$\Delta m_B \equiv M_H - M_L, \quad \Delta\Gamma_B \equiv \Gamma_H - \Gamma_L. \tag{B10}$$

Note that here Δm_B is positive by definition, while the sign of $\Delta\Gamma_B$ is to be experimentally determined. The average mass and width are given by

$$m_B \equiv \frac{M_H+M_L}{2}, \quad \Gamma_B \equiv \frac{\Gamma_H+\Gamma_L}{2}. \tag{B11}$$

It is useful to define dimensionless ratios x and y:

$$x \equiv \frac{\Delta m_B}{\Gamma_B}, \quad y \equiv \frac{\Delta\Gamma_B}{2\Gamma_B}. \tag{B12}$$

Solving the eigenvalue equation gives

$$(\Delta m_B)^2 - \frac{1}{4}(\Delta\Gamma_B)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \qquad \Delta m_B \Delta\Gamma_B = 4\mathcal{R}e(M_{12}\Gamma_{12}^*). \tag{B13}$$

All CP-violating observables in B and $\overline{B}$ decays to final states f and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of A_f, $\overline{A}_f$, $A_{\overline{f}}$ and $\overline{A}_{\overline{f}}$, together with, for neutral-meson decays only, q/p. CP violation in charged-meson decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while CP violation in neutral-meson decays is complicated by $B^0 \leftrightarrow \overline{B}^0$ oscillations and depends, additionally, on $|q/p|$ and on $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$.

For neutral D, B, and B_s mesons, $\Delta\Gamma/\Gamma \ll 1$ and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|B^0\rangle$ or $|\overline{B}^0\rangle$ after an elapsed proper time t as $|B^0{}_{\rm phys}(t)\rangle$ or $|\overline{B}^0_{\rm phys}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, we obtain

$$\begin{aligned} |B^0{}_{\rm phys}(t)\rangle &= g_+(t)\,|B^0\rangle - \frac{q}{p}\, g_-(t)|\overline{B}^0\rangle, \\ |\overline{B}^0_{\rm phys}(t)\rangle &= g_+(t)\,|\overline{B}^0\rangle - \frac{p}{q}\, g_-(t)|B^0\rangle\ , \end{aligned} \tag{B14}$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right). \tag{B15}$$

One obtains the following time-dependent decay rates:

$$\begin{aligned} \frac{d\Gamma[B^0{}_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \Big(|A_f|^2 + |(q/p)\overline{A}_f|^2\Big)\cosh(y\Gamma t) + \Big(|A_f|^2 - |(q/p)\overline{A}_f|^2\Big)\cos(x\Gamma t) \\ &+\ 2\,\mathcal{R}e((q/p)A_f^*\overline{A}_f)\sinh(y\Gamma t) - 2\,\mathcal{I}m((q/p)A_f^*\overline{A}_f)\sin(x\Gamma t)\ , \qquad \text{(B16)} \\ \frac{d\Gamma[\overline{B}^0_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \Big(|(p/q)A_f|^2 + |\overline{A}_f|^2\Big)\cosh(y\Gamma t) - \Big(|(p/q)A_f|^2 - |\overline{A}_f|^2\Big)\cos(x\Gamma t) \\ &+\ 2\,\mathcal{R}e((p/q)A_f\overline{A}_f^*)\sinh(y\Gamma t) - 2\,\mathcal{I}m((p/q)A_f\overline{A}_f^*)\sin(x\Gamma t)\ , \qquad \text{(B17)} \end{aligned}$$

where $\mathcal{N}_f$ is a common normalization factor. Decay rates to the CP-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f = \mathcal{N}_{\overline{f}}$ and the substitutions $A_f \to A_{\overline{f}}$ and $\overline{A}_f \to \overline{A}_{\overline{f}}$ in Eqs. (B16,B17). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $B \leftrightarrow \overline{B}$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$

are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (B16,B17) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of phase-space variables. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

One possible manifestation of CP-violating effects in meson decays [27] is in the interference between a decay without mixing, $B^0 \to f$, and a decay with mixing, $B^0 \to \overline{B}^0 \to f$ (such an effect occurs only in decays to final states that are common to B^0 and $\overline{B}^0$, including all CP eigenstates). It is defined by

$$\mathcal{I}m(\lambda_f) \neq 0 \, , \tag{B18}$$

with

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f} \, . \tag{B19}$$

This form of CP violation can be observed, for example, using the asymmetry of neutral meson decays into final CP eigenstates f_{CP}

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}\,] - d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}\,]}{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}\,] + d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}\,]} \, . \tag{B20}$$

For $\Delta\Gamma = 0$ and $|q/p| = 1$ (which is a good approximation for B mesons), $\mathcal{A}_{f_{CP}}$ has a particularly simple form [28–30]:

$$\begin{aligned} \mathcal{A}_f(t) &= S_f \sin(\Delta m t) - C_f \cos(\Delta m t), \\ S_f &\equiv \frac{2\,\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2}, \quad C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \, , \end{aligned} \tag{B21}$$

Consider the $B \to f$ decay amplitude A_f, and the CP conjugate process, $\overline{B} \to \overline{f}$, with decay amplitude $\overline{A}_{\overline{f}}$. There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in A_f and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model, these phases occur only in the couplings of the $W^{\pm}$ bosons and hence are often called "weak phases". The weak phase of any single term is convention dependent. However, the difference between the weak phases in two different terms in A_f is convention independent. A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Their origin is the possible contribution from intermediate on-shell states in the decay process. Since these phases are generated by CP-invariant interactions, they are the same in A_f and $\overline{A}_{\overline{f}}$. Usually the dominant rescattering is due to strong interactions and hence the designation "strong phases" for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The 'weak' and 'strong' phases discussed here appear in addition to the 'spurious' CP-transformation phases of Eq. (B3). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any CP violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution a_i to A_f in three parts: its magnitude $|a_i|$, its weak phase ϕ_i, and its strong phase δ_i. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$\begin{aligned} A_f &= |a_1| e^{i(\delta_1+\phi_1)} + |a_2| e^{i(\delta_2+\phi_2)}, \\ \overline{A}_{\overline{f}} &= |a_1| e^{i(\delta_1-\phi_1)} + |a_2| e^{i(\delta_2-\phi_2)}. \end{aligned} \qquad \text{(B22)}$$

Similarly, for neutral meson decays, it is useful to write

$$M_{12} = |M_{12}| e^{i\phi_M} \quad , \quad \Gamma_{12} = |\Gamma_{12}| e^{i\phi_\Gamma} \ . \qquad \text{(B23)}$$

Each of the phases appearing in Eqs. (B22,B23) is convention dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$ and $\phi_M + \phi_1 - \overline{\phi}_1$ (where $\overline{\phi}_1$ is a weak phase contributing to $\overline{A}_f$) are physical.

In the approximations that only a single weak phase contributes to decay, $A_f = |a_f| e^{i(\delta_f+\phi_f)}$, and that $|\Gamma_{12}/M_{12}| = 0$, we obtain $|\lambda_f| = 1$ and the CP asymmetries in decays to a final CP eigenstate f [Eq. (B20)] with eigenvalue $\eta_f = \pm 1$ are given by

$$\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_f)\ \sin(\Delta m t) \ \text{ with } \ \mathcal{I}m(\lambda_f) = \eta_f \sin(\phi_M + 2\phi_f). \qquad \text{(B24)}$$

Note that the phase so measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from $\mathcal{I}m(\lambda_f)$.

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